Homework 1

1. Define a time-dependent equation for the angular flux associated with a beam of particles that travels along the x-axis in the positive x-direction with speed v. We denote this angular flux by $\psi^+(x)$. It has units of (p/sec). Assume an absorption cross-section, σ_a, with units of cm^{-1}, and an inhomogeneous source function, Q^+, with units of $p/(cm - sec)$.

   - Solve the steady-state version of this equation on $[0, x_0]$, with boundary condition, $\psi^+(0) = 1$ and $Q = 0$.
   - Solve the steady-state version of this equation on $[0, x_0]$, with boundary condition, $\psi^+(0) = 0$ and $Q = 1$.

2. Define a time-dependent equation for the the angular flux associated with a beam of particles that travels along the x-axis in the negative x-direction with speed v. We denote this angular flux by $\psi^-(x)$. It has units of (p/sec). Assume an absorption cross-section, σ_a, with units of cm^{-1}, and an inhomogeneous source function, Q^-, with units of $p/(cm - sec)$. Also assume that $Q^- = Q^+$.

3. Define a scattering cross section, σ_s, with units of cm^{-1}, such that when a particle scatters, it transfers to the $+$ direction half of the time and the $-$ direction the other

half of the time no matter what its initial direction. Include the scattering interaction in the equations for ψ^+ and ψ^-.

4. Define the scalar flux as follows, $\phi \equiv \psi^+ + \psi^-$, and define the current as follows, $J \equiv \psi^+ - \psi^-$.

   - Add the equations for ψ^- and ψ^+, and call the resulting equation, "Equation 1".
   - Re-express Eq. 1 in terms of the scalar flux and current. You should now have a balance equation for the two-stream system. Remember that the scattering terms should not be present in this equation.
   - Subtract the equation for ψ^- from the equation for ψ^+, and call the resulting equation, "Equation 2".
   - Re-express Eq. 2 in terms of the scalar flux and current.
   - Set the time-derivative of the current to zero in Eq. 2, and then manipulate the equation to solve for the current in terms of the scalar flux. You should get a version of Fick's law, which states that the current is proportional to a coefficient times the negative of the gradient of the scalar flux.
   - Use this relationship to eliminate the current from Eq. 1. You should get a time-dependent diffusion equation. Thus, the scalar flux associated the simplified two-stream model satisfies a time-dependent diffusion equation if the time-derivative

of the current is assumed to be zero. Furthermore, for steady-state calculations, the scalar flux rigorously satisfies a diffusion equation.

5. Solve the following equation

$$\mu \frac{\partial \psi}{\partial x} + \sigma_a \psi = \frac{Q_o}{4\pi} \,,$$

subject to the following boundary conditions and parametric definitions.

$$\psi(x_L, \mu) = 0.0 \quad , \text{for } \mu > 0.$$

$$\psi(x_R, \mu) = 0.0 \quad , \text{for } \mu < 0.$$

$$Q_o(x) = 1.0 \quad , \frac{p}{cm^3 - sec}.$$

$$x_L = 0.0 \quad , cm.$$

$$x_r = 1.0 \quad , cm.$$

$$\sigma_a = 1.0 \quad , cm^{-1}.$$

- What is the rate at which particles in direction μ leave the slab per cm^2 of surface area?
- What is the rate at which particles in direction μ come into the slab per cm^2 of surface area?
- What is the rate at which particles in direction μ are absorbed in the slab per cm^2 of surface area?